LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2025**

PST1MC01 – ADVANCED DISTRIBUTION THEORY

Date: 23-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Define the following**
a)	Cauchy distribution.
b)	Multinomial distribution.
c)	Non-central chi-square distribution.
d)	Distribution of sample range and sample midrange.
e)	Idempotent matrix.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	The mean of beta distribution of first kind is_____________
b)	The MGF of bivariate normal distribution is____________
c)	The variance of non-central F-distribution is_______________
d)	The asymptotic distribution of the k^{th} order statistic from a normal distribution is _______________
e)	If $X\sim N(0,1)$ and A and B are real $n \times n$ symmetric matrices, then $COV[X'AX, X'BX]$ is _______
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Establish the r^{th} factorial moments of the hypergeometric distribution and hence find its variance.
4	Derive the marginal and conditional distributions of the multinomial distribution.
5	Derive the mean and variance of non-central 'F' distribution.
6	Show that in odd samples of size n from U[0,1] population, the mean and variance of the distribution of median are ½ and 1/[4(n+2)] respectively.
7	Derive the MGF of quadratic form.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	(i)Prove that $P(Y \leq x \mid X \geq a) = P(X \leq x)\ for\ all\ x, where\ Y = x - a$ for an exponential distribution.

	(ii)If two normal universes A and B have the same total frequency but the standard deviation of universe A is k times that of the universe of B, show that maximum frequency of universe A is (1/k) times that of universe B. **(9+3.5)**
9	If $X_1, X_2, \dots, X_n$ be iid $N(0, \sigma^2), \sigma > 0$. Define $X = (X_1, X_2, \dots, X_n)'$and $Q = X'AX$ with $\rho(A) = r$. Then $\frac{Q}{\sigma^2}$ is distributed as chi-square distribution with r degrees of freedom iff A is idempotent.
10	Derive the p.d.f of non-central 't' distribution.
11	If X and Y are independent Gamma variates with parameters μ and v respectively, then show that the variables U = X+Y, $Z = \frac{X}{Y}$ are independent and that U is a $\gamma(\mu + v)$ variate and Z is a $\beta_2(\mu + v)$ variate.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Establish the p.d.f of n^{th} order statistic.
13	Derive the probability generating function, marginal and conditional distributions for bivariate poisson distribution.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	State and prove Cochran's theorem.
15	(i) Let $X \sim N_2(\mu, \Sigma)$ where $\mu = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \Sigma = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$. (a) Find the distribution of $2X_1 + 3X_2$. (b). Find the distribution of $(3X_1 - X_2, 6X_1 + 5X_2)$ and (c). Examine if $X_1 + X_2$ and $2X_1 - 3X_2$ are independent. **(3+4+3)** (ii) Derive the recurrence relation for the moments of normal distribution. **(10)**

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